Uniform vs. Discriminatory Auctions with Variable Supply – Experimental Evidence*

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Abstract

In the variable supply auction considered here, the seller decides how many customers with unit demand to serve after observing their bids. Bidders are uncertain about the seller's cost. We experimentally investigate whether a uniform or a discriminatory price auction is better for the seller in this setting. Exactly as predicted by theory, it turns out that the uniform price auction produces substantially higher bids, and consequently yields higher revenues and profits for the seller. Furthermore, again as predicted by theory, the uniform price auction yields a higher number of transactions, which makes it also the more efficient auction format.

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1 Introduction

A *variable supply* auction is an auction in which the number of items sold is not fixed in advance but can be determined by the seller in a profit maximizing way after seeing the submitted bids. To capture bidders' uncertainty about supply, let us assume that the seller's production cost (or his reserve price) for each object is his private information.[1] Although variable supply auctions with private reservation values of sellers are actually quite frequent in reality,[2] to our knowledge, this important institutional feature of auctions has so far not been explored experimentally. Seller's ability to determine supply ex post adds important new aspects to the analysis of the standard auctions. Since bidders have to anticipate the seller's quantity choice, their incentives and bidding behavior are influenced in a complex way. Thus, variable supply auctions have intriguing game theoretic qualities and it seems natural to wonder about the revenue effects and the efficiency of such auctions.

In this paper, we present an experimental comparison of different auction formats with variable supply. In particular, we consider the two most widely used auction formats for variable supply auctions, the uniform price auction and the discriminatory price auction. Based on the theoretical work of Damianov and Becker (2008), we first derive the theoretical prediction that the uniform price auction outperforms the discriminatory price auction from the seller's viewpoint and in terms of efficiency. We also extend those theoretical predictions to the case of heterogeneous bidders and the case of risk averse bidders and show that the qualitative results remain unchanged. Then, we experimentally test those predictions in the laboratory. Exactly as predicted by theory, it turns out that the uniform price auction produces substantially higher bids, and consequently yields higher revenues and profits for the seller. Furthermore, as predicted by theory, the uniform

[1] As an example consider a "secret reserve price" on eBay.

[2] See Loxley and Salant (2004, p. 224), Keloharju et al. (2005), and Busaba et al. (2001) for examples of variable supply auctions for electricity, Treasury bills, and IPOs, respectively. In particular, the discriminatory pricing rule in variable supply auctions is used by the Treasury departments of Sweden, Switzerland, Finland, Germany, and Mexico. The treasuries of Italy and Norway use uniform pricing.

price auction yields a higher number of transactions, which makes it also the more efficient auction format.

In the discriminatory auction, each bidder has to pay his bid if served by the seller. In the uniform auction, all bidders served by the seller pay the same price, which equals the lowest bid served.[3] If several of the bids are above marginal cost, the seller faces a trade–off between serving a smaller number of buyers and receiving a higher price, or serving a larger number of buyers at a lower price.

The theoretical literature on variable supply auctions starts with Lengwiler (1999) who assumes that buyers are uncertain about the (constant) marginal cost of the seller, and allows these bidders to announce quantities at two exogenously given price levels. Lengwiler describes the equilibria of the uniform and the discriminatory auctions, and concludes that the two auctions cannot be ranked in terms of seller revenues and trade surplus. Several subsequent contributions (see Back and Zender, 2001; Damianov, 2005; McAdams, 2007) focus only on the uniform price auction and conduct their analyses in a complete information framework in which bidders announce entire demand schedules. The major conclusion of these papers is that the seller will receive higher bids by retaining the right to reduce supply ex post.

The only experimental paper that considers an auction with variable supply is by Sade et al. (2006). In their setting there is no uncertainty about the seller's cost. The focus of the paper is on the possibility of collusion and bidders are therefore allowed to communicate before each round of bidding. They find that a variable supply uniform price auction generates higher revenue than the corresponding auction with fixed supply although the difference is not statistically significant. They also find that the discriminatory fixed supply auction is more susceptible to collusion and therefore yields the lowest revenues for the seller.

Damianov and Becker (2008) analyze variable supply auctions for the

[3] In the literature one sometimes finds an alternative version in which the price equals the highest bid not served. However, in practice this version does not seem to be in much use; e.g. most treasury auctions use our version.

case of n risk neutral bidders and a general distribution of production cost. They show that in every symmetric equilibrium, bidders submit higher bids in the uniform than in the discriminatory auction. In the case of two bidders, this result holds even for all rationalizable bids. Since both bids in the uniform auction are higher than those in the discriminatory one, the uniform auction generates higher revenues and is more profitable for the seller. The uniform auction is also more efficient as it generates a higher trade volume.[4] This may seem counterintuitive as in the discriminatory auction all bids above marginal cost are being served whereas in the uniform auction, some bids above marginal cost are rejected by the seller.

For simplicity, the theoretical analysis, and consequently also our experimental design, is based on a number of fairly restrictive assumptions. In particular, the value of the auctioned goods is commonly known and bidders have unit demand. However, already for this simplified setting the theoretical treatment of the uniform auction is far from trivial. Future theoretical and experimental work needs to generalize the current setting but we see our contribution as a first step towards the understanding of variable supply auctions from which we or the profession can broaden our understanding in various directions.

The remainder of this paper is structured as follows. In Section 2 we introduce the auction rules and derive theoretical predictions. We consider an auction with two bidders and a uniform distribution of the seller's production cost. We obtain clear and testable predictions for the difference between the discriminatory price auction and the uniform price auction. In particular, the uniform auction should yield significantly higher bids, higher revenues and profits for the seller, and a higher number of transactions. We also consider the case of bidders with heterogeneous values and the case of risk averse bidders and confirm that the main results remain unchanged.

Section 3 introduces the experimental design. In Section 4 we present the experimental results, which are quite close to the theoretical predictions. As predicted, bids, profits, revenues, and number of transactions are significantly higher in the uniform auction. Subjects seem to learn how to bid

[4] The latter result holds for all weakly convex distribution functions of production cost.

in these auctions as the experimental data are even closer to the theoretical predictions in the second half of the experiment. Other factors, like the extent of experience with auctions such as eBay, do not seem to matter for the bidding behavior of our subjects. In Section 5 we conclude. Some proofs and the instructions for the experiment are collected in an appendix.

2 Auction rules and theoretical predictions

In this section we introduce the auction rules and derive theoretical predictions. A monopoly seller offers multiple items of a good to two prospective buyers, $i = 1, 2$. Buyers are risk neutral and have a demand for one item with a common value of v for the item. Buyers simultaneously submit sealed bids b_i to the seller from a discrete price grid $B = \{0, \Delta, 2\Delta, \ \ldots \ , M\Delta\}$. We assume that $\Delta = v/(100n)$, where n is an arbitrary integer, $n \geq 1$, which guarantees that v and $v/2$ are on the grid and that the grid is sufficiently fine to approximate a continuous strategy space. To allow for bids up to 10% above one's value, we set $M = 110n$.[5]

The seller has constant marginal cost of production c for each item and c is uniformly distributed over the interval $[0, v]$. The realization of the cost is private information of the seller. The seller can decide on the number of items to sell after observing the bids. Thus, with two bidders, the seller can choose to sell 0, 1, or 2 items in each auction and will always choose a profit maximizing option. If the seller is indifferent between two quantities, we will assume that the seller will sell the higher quantity. We make this assumption only for completeness. It does not play a role in the analysis because such events occur with probability zero (marginal cost c is drawn from a continuous distribution).

The two auction formats are as follows.

- In the *discriminatory* price auction, each bidder has to pay his bid if served by the seller. A rational seller will sell one unit to each bidder who bids at or above his marginal cost.

[5] Allowing for even higher bids would not change any of our theoretical results.

- In the *uniform* price auction, the auction price equals the lowest bid that is served by the seller. The seller maximizes his profits by either selling 0 (if both bids are below marginal cost), one or two units. Thus, the seller will sell one unit rather than two if $b_1 - c > 2(b_2 - c)$, where $b_1 > b_2$ are the bids. If $b_1 - c \leq 2(b_2 - c)$, the seller will serve both bidders.

How will rational bidders behave in these two auction games? In the following analysis we always assume that the seller chooses a profit maximizing supply quantity and concentrate on the reduced game between bidders.

The case of the discriminatory auction is a straightforward decision problem for each bidder. Since the seller serves all bids (weakly) above marginal cost and the price a bidder pays equals his bid, optimal strategies are independent of the other player. The expected payoff of bidder i is given by the payoff conditional on winning times the probability of winning, that is $R_i^D(b_1, b_2) = (v - b_i)b_i/v$. The optimal bid is $b_i^D = v/2$. Since this is independent of the other player's behavior, we have

Proposition 1 *In the discriminatory auction bidding $v/2$ is a strictly dominant strategy.*

Consequently, the expected revenue of the seller is v and the expected number of transactions per round is 1.

The case of the uniform price auction is more complex. Whether a bidder will be served and the price he has to pay depend in general on both bids b_1 and b_2. Let us first specify the payoff of player 1.[6] This payoff depends on whether player 1 bids lower or higher than player 2, and we will discuss in turn these two possibilities.

Case 1: $b_1 < b_2$. Bidder 1 will be served only if the seller finds it profitable to sell both units at the price of b_1. This is the case when $2(b_1 - c) \geq b_2 - c$ or, equivalently, $c \leq 2b_1 - b_2$. Hence, bidder 1 will be served with probability $\text{Prob}(c \leq 2b_1 - b_2)$, and the expected payoff of bidder 1 is given

[6] The payoff of player 2 can be determined analogously as the game is symmetric.

by the expression

$$R_1^U(b_1, b_2) = (v - b_1)\,\text{Prob}(c \le 2b_1 - b_2).$$

Case 2: $b_1 \ge b_2$. If both bidders are served, they will pay b_2. If only bidder 1 is served, he will pay b_1. The seller finds it profitable to sell both units if $2(b_2 - c) \ge b_1 - c$ or, equivalently, $c \le 2b_2 - b_1$. Hence, bidder 1 will receive an item at the price of b_2 with a probability of $\text{Prob}(c \le 2b_2 - b_1)$. The seller finds it profitable to sell only one unit at the price of b_1 if $2b_2 - b_1 < c \le b_1$, and the probability for this event is $\text{Prob}(2b_2 - b_1 < c \le b_1)$. Hence, the expected payoff of bidder 1 in this case is given by the expression

$$R_1^U(b_1, b_2) = (v - b_2)\,\text{Prob}(c \le 2b_2 - b_1) + (v - b_1)\,\text{Prob}(2b_2 - b_1 < c \le b_1).$$

Notice that for $2b_2 < b_1$ we have $\text{Prob}(2b_2 - b_1 < c \le b_1) = \text{Prob}\ (c \le b_1)$, and $\text{Prob}(c \le 2b_2 - b_1) = 0$. Observe also that $\text{Prob}(c \le 2b_1 - b_2) = 0$ for $b_1 < b_2/2$. Therefore the expected payoff of bidder 1 in Cases 1 and 2 can be summarized by the following expression
$R_1^U(b_1, b_2) =$

$$\begin{cases} (v - b_1)\,\text{Prob}(c \le b_1) & \text{for } 2b_2 < b_1, \\ (v - b_2)\,\text{Prob}(c \le 2b_2 - b_1) & \\ \quad + (v - b_1)\,\text{Prob}(2b_2 - b_1 < c \le b_1) & \text{for } b_2 \le b_1 \le 2b_2, \\ (v - b_1)\,\text{Prob}(c \le 2b_1 - b_2) & \text{for } b_2/2 \le b_1 < b_2, \\ 0 & \text{for } b_1 < b_2/2. \end{cases} \qquad (1)$$

Note first that no symmetric, pure strategy equilibrium exists for the uniform auction. The intuition for why no pair (b, b) of bids lower than $v - 2\Delta$, in particular not $(v/2, v/2)$, can be an equilibrium, is as follows. Suppose $b_1 = b_2 < v - 2\Delta$ and bidder 1 considers increasing his bid marginally to $b_1 + \Delta$. If $c > b_1 + \Delta$, then no trade takes place anyway. If $c \le b_2 - \Delta$, bidder 1 is equally well off as without the increase: in both cases he receives an item and pays b_2 since the seller finds it profitable to sell two units. If $b_2 < c \le b_1 + \Delta$, then bidder 1 will obtain the item and receive payoff $v - b_1 - \Delta > \Delta$. Without the increase, bidder 1's payoff would have been 0. This event happens with a probability of Δ/v (marginal cost c is uniformly distributed over the interval $[0, v]$, and $\Delta = v/n$). Finally, if $b_2 - \Delta < c \leq b_2$, bidder 1 will receive the item but will pay Δ more than necessary. Again, this event happens with a probability of Δ/v. Hence, in total there is a *gain* from the deviation of $v - b_1 - \Delta > \Delta$ with a probability of Δ/v and a *loss* of Δ with the same probability of Δ/v. The expected gain exceeds the expected loss and the deviation is profitable.

In the case $b_1 = b_2 \geq v - 2\Delta$, it is profitable for bidder 1 to incrementally lower his bid. Indeed, by bidding $b_1 - \Delta$ bidder 1 will be served with a probability of $\min\{(b_1 - 2\Delta)/v, 1\} \geq (v - 4\Delta)/v$ and will need to pay Δ less. By lowering his bid, bidder 1 reduces the probability of receiving an item by no more than $2\Delta/v$, in which case he will forego a surplus of $v - b_1 \leq 2\Delta$. In sum, by lowering his bid from b_1 to $b_1 - \Delta$ bidder 1 will gain at least $\Delta \cdot (v - 4\Delta)/v$ and will lose no more than $2\Delta \cdot 2\Delta/v$. Since $\Delta \leq v/100$, this deviation is profitable.

As it turns out, the number of equilibria of the uniform auction game is so large that no specific equilibrium can have much predictive power. Fortunately, much progress can be made by considering rationalizable strategies, which provide a relatively sharp prediction. Of course, the support of all Nash equilibria of the reduced game between the two bidders belongs to the set of rationalizable strategies. Note that in finite two–player games the set of rationalizable strategies is identical to the set of strategies that survive the iterative elimination of strictly dominated strategies (see Pearce, 1984). We will therefore use those concepts interchangeably in this paper.

The following proposition characterizes the set of rationalizable strategies. The lower bound was already derived by Damianov and Becker (2008) as a special case of a more general setting. However, in the appendix we provide a new and self–contained proof. The upper bound of rationalizable bids is new.

Proposition 2 *In the uniform price auction, strategies below* $0.66v$ *and above* $0.84v$ *do not survive the iterated elimination of strictly dominated strategies.*

Proof. See Appendix. ■

In Figure 1 the set of rationalizable strategies for both players is shown as the square between the $0.66v$- and the $0.84v$-lines. Also shown are the best response correspondences defined on the discrete grid (see the Appendix for details). At the intersections of the best response correspondences there are two asymmetric pure strategy Nash equilibria $(b_i, b_{-i}) = (0.77v, 0.69v)$, $i = 1, 2$.

With the help of the numerical algorithm *Gambit* (McKelvey et al., 2007), we were able (for a grid with size $0.01v$) to calculate a symmetric mixed strategy equilibrium but also more than a thousand asymmetric mixed strategy equilibria with support in $[0.69v, 0.81v]$. Given this multiplicity of equilibria, it is best to focus on the set of rationalizable strategies, which necessarily contains the support of all Nash equilibria.

To develop an intuition for why bidders submit higher bids in the uniform price auction let us compare the effects of increasing a player's bid in the two auction formats. While in the discriminatory auction a higher bid ultimately leads to a higher transaction price for that bidder, this is not always the case for the uniform price auction. In the uniform price auction, by raising his bid, a bidder is able to raise his chances of winning, but not necessarily the price he pays because in some cases the price will be determined by the lower bid. It can easily be seen that as long as both bids do not exceed v, the bidder who submits the higher bid has a higher expected payoff. The higher bidder receives an item in all cases in which the lower bidder is served, and pays the same price in this case, but the higher bidder has an additional chance of winning the item at a price below v. This payoff structure creates an opportunity for bidders to "free ride" on their lower bidding counterparts. It promotes competition among bidders by providing additional incentives for bidders to bid high.

2.1 Asymmetric valuations

Our analysis so far assumed that bidders have the same valuation for the item. Our next statement relaxes this assumption and demonstrates that the established revenue and efficiency ranking result remains valid even if

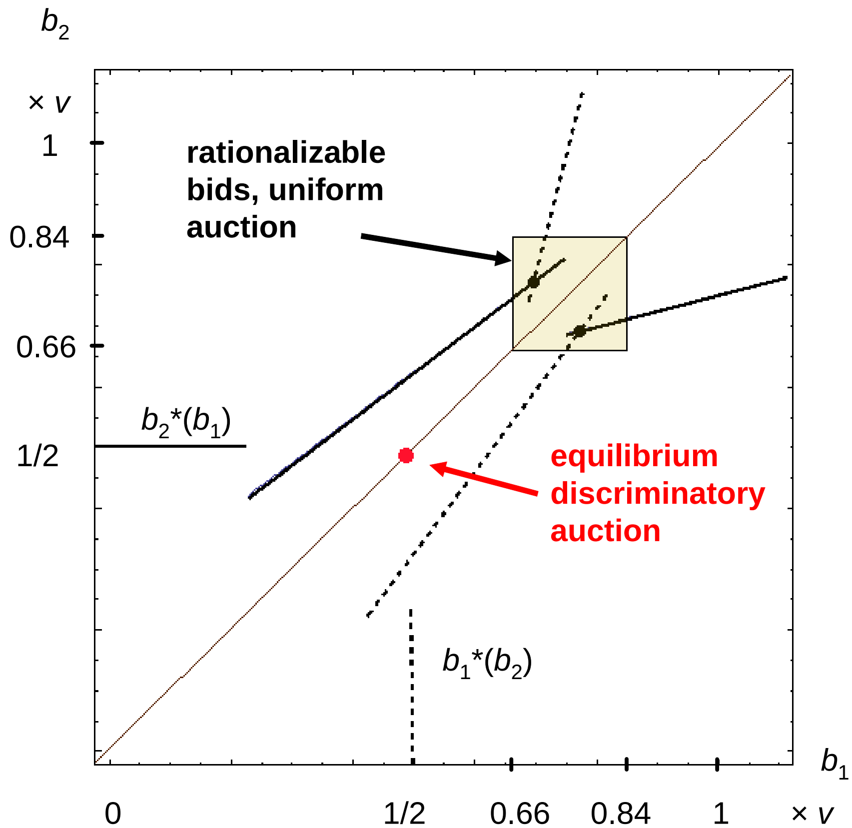

Figure 1: Set of rationalizable bids (shaded square) and pure strategy best reply correspondences for the uniform price auction.

Note: The two asymmetric pure strategy equilibria of the uniform auction are at the intersections of the best reply correspondences (which take values on the fine discrete grid). The unique equilibrium of the discriminatory auction at $(v/2, v/2)$ is shown for comparison.

we allow for some asymmetry in the valuations of the bidders.

Corollary 1 *If the valuations of the bidders* v_1 *and* v_2 *belong to the interval* $[0.78v, v]$ *and are divisible by* $0.02v$*, all rationalizable bids in the uniform price auction are strictly higher than the dominant strategy bids in the discriminatory auction.*

Proof. This corollary follows almost directly from the proofs of Propositions 1 and 2. Let us assume without loss of generality that $v_1 \leq v_2$. Following the arguments presented in Proposition 1, it is easily seen that in the discriminatory auction the dominant strategy bids for bidder 1 and bidder 2 are given by $v_1/2$ and $v_2/2$, respectively. As the valuations are divisible by $0.02v$, the bids $v_1/2$ and $v_2/2$ belong to the grid.

For the uniform price auction, consider the following argument. Inspection of the payoff function (1) shows that if $R_1^U(b_1, b_2; v_1)$ is increasing in b_1 for given b_1, b_2 and v_1, then it is also increasing in b_1 for any $v_1' \geq v_1$. Thus, if a bid is dominated by a higher bid from the viewpoint of bidder 1, it will also be dominated from the viewpoint of bidder 2. We can now apply the procedure presented in the proof of Proposition 2 and iteratively eliminate all bids below $0.66v_1$ for both bidders. Hence, bids below $0.66 \cdot 0.78v$ are not rationalizable. This lower bound is strictly higher than $0.51v$ and hence higher than the dominant strategy bids of both bidders in the discriminatory auction. ■

2.2 The case of risk averse bidders

In this subsection we will briefly address the case of risk averse bidders. Recall that a utility function $\tilde{u}(x)$ is strictly more risk averse (or has a higher degree of risk aversion) than a utility function $u(x)$ if there exists an increasing and strictly concave function $\psi(\cdot)$ such that $\tilde{u}(x)$ is a concave transformation of $u(x)$, $\tilde{u}(x) = \psi(u(x))$.[7]

The next proposition shows that for a sufficiently fine grid in both auction formats the rationalizable bids are increasing in the risk aversion of

[7] See e.g. MasColell et al. (1995, p. 191) for details and various equivalent formulations of *more-risk-averse-than* relations.

bidders. In particular, bids are higher than in the risk neutral case. More importantly, with risk aversion, rationalizable bids in the uniform auction remain strictly above those in the discriminatory auction.

Proposition 3 *For n sufficiently large (i.e. for sufficiently small grid size $\Delta = v/(100n)$, the following holds.*

1. *The dominant strategy in the discriminatory auction is strictly increasing in the degree of risk aversion. In particular, risk averse bidders bid higher than risk neutral bidders: $b^D > v/2$.*

2. *The lower bound on rationalizable bids in the uniform price auction is strictly increasing in the degree of risk aversion. In particular, this lower bound is higher for risk averse bidders than for risk neutral bidders: $\underline{b}^U > 2v/3$.*

3. *The lower bound on rationalizable bids in the uniform auction is strictly above the dominant strategy for the discriminatory auction also when bidders are risk averse: $\underline{b}^U > b^D$.*

Proof. See Appendix. ■

Thus, qualitatively the predictions for the risk averse case are the same as those for the risk neutral case: the uniform auction elicits higher bids in any equilibrium. However, since the predictions obviously depend on the exact form of the utility function, we will in the following use the risk neutral case as a benchmark for our experimental results.

3 Experimental design

In each auction two bidders had the opportunity to bid for a good, which had a common redemption value of $v = 200$ cents (= 2 euros) for both of them. All payoffs were denominated in actual (euro-)cents such that no exchange rate was necessary.

The number of auction periods was 20 in all sessions and this was commonly known. The seller (whose role was played by the computer program) in each round had constant marginal cost c of production for the objects. In each round the marginal costs (in cents) were drawn from an i.i.d. uniform distribution on $\{0, 0.1, 0.2, ..., 199.9, 200.0\}$ and the realizations were unknown to bidders at the time of bidding. To improve the pairwise comparison across treatments, these cost realizations were drawn ahead of time and the same set of 20 cost realizations was used for all matching groups and for both of our treatments.[8]

The seller – played by a computer program – could choose to sell 0, 1 or 2 objects in each auction and would always choose the profit maximizing option. This rule was known to subjects. Each bidder submitted a sealed bid for one object in each round. Bids could be any amount (in cents) from the set $\{0, 0.1, 0.2, ..., 219.9, 220.0\}$, which corresponds to $n = 2000$ and $\Delta = 0.1$ Euro-cents in the notation from the previous section.

At the beginning of the experiment each bidder received an endowment of 400 cents, which made it impossible to lose money in the experiment. After each auction period, bidders were informed about the size of the two bids, how many units were sold (0,1, or 2), what the payoffs for the two bidders were, what the production cost of the seller was, and their cumulative payoff up to that period. Subjects were asked to record all this information on a "record sheet".

There were two treatments.

- In the *discriminatory* price treatment, the computer would sell a unit to each bidder who bid above this period's marginal cost and subjects had to pay their respective bids.
- In the *uniform* price treatment, the auction price equaled the lowest bid that was served by the seller. The seller maximized his profits by either selling 0 (if both bids were below marginal cost), one or two units.

The auction rules were explained to subjects in detail and with examples.

[8]For rounds 1 through 20 the realizations of costs were 133.4, 86.3, 107.6, 93.4, 12.2, 50.3, 174.7, 117.9, 160.6, 62.3, 20.5, 189.8, 137.1, 82.4, 139.5, 12.6, 186.2, 5.8, 67.4, and 133.2, respectively, with a mean value of 98.66.

Several test questions were conducted to make sure subjects understood the auction rules. Instructions (see Appendix B) were written on paper and distributed in the beginning of each session. When subjects were familiar with the rules, we started the first round.

The experiments were conducted in June 2007 in the computer lab of the SFB 504, Mannheim. All subjects were recruited via the ORSEE online recruiting system (Greiner, 2004). For the experiment, we used the z–tree software package provided by Fischbacher (2007). In each session 18 subjects participated, constituting three matching groups of six bidders. When entering the lab, subjects were randomly allocated to computer terminals in the lab such that they could not infer with whom they would interact in a group of six. Each group was independent of the others. Matching within groups was done randomly in each round such that no subject could infer who the other bidder was. For both treatments we had six independent matching groups of subjects – making a total of 72 ($= 2 \times 6 \times 6$) subjects who participated in the experiment.

Subjects were paid the sum of their earnings from all auctions. The average payoff was about 12.40 euros. Experiments lasted less than 60 minutes including instruction time.

Table 1 summarizes the predictions in the risk neutral case for the two auction formats given the parameters used in the experiment ($v = 200$). In the discriminatory auction, there is a strictly dominant strategy of bidding 100. For the uniform price auction, the rationalizable bids are given by Proposition 2 as bids in the interval $[133.3, 170]$. The upper and the lower limits for profits, revenues, and number of transactions are obtained by using the lowest, respectively highest, rationalizable bids for both players. For example, when both bidders bid 133.3, then expected revenues are 133.3 times the expected number of transactions, i.e. $133.3 \times 4/3 = 177.7$. Note that the higher profits in the uniform auction follow from the fact that in *any* equilibrium *both* bids in the uniform auction are higher than those in the discriminatory auction.

The main prediction is clearly that the uniform auction dominates the discriminatory auction from the seller's viewpoint. Since all rationalizable

Table 1: Theoretical predictions for parameters used in the experiment

Auction	rationalizable bids	expected seller profits	expected seller revenues	expected # of transactions
Uniform	$[133.3, 170]$	$[88.8, 144.5]$	$[177.7, 289.0]$	$[4/3, 1.7]$
Discriminatory	100	50	100	1

Note: All measures are calculated per round and for both bidders. The two asymmetric pure strategy equilibria in the uniform auction are at (154,138) and (138,154). Predictions apply to the risk neutral case.

bids are strictly higher in the uniform auction, seller's profits and revenues exceed those in the discriminatory auction. Moreover, the uniform auction is more efficient because the expected number of transactions is higher.

The last result seems counterintuitive since in the discriminatory auction all bids above marginal cost are being served while in the uniform auction, some bids above marginal cost are rejected by the seller.[9] Nevertheless, bids are so much higher in the uniform auction treatment that they result in a significantly higher number of transactions.

4 Experimental results

We can now compare the theoretical predictions summarized in Table 1 to the data from our experiment. Table 2 shows mean bids, the means of the higher and of the lower of the two bids in a given match, mean seller profits and revenues, and the average number of transactions per round. Mean seller costs are shown for comparison. The table presents those measures separately for each treatment and separately for all rounds and the final 10 rounds, respectively.

Mean values of bids, revenues, profits, and transactions are quite close to the theoretical predictions for the discriminatory auction. Data for the uni-

[9] A referee pointed out that this result may not be counterintuitive after all. The reason is that while there is monopoly power of the seller in the uniform auction, there is monopoly power of the bidders in the discriminatory auction. A priori, it is not clear which effect should dominate.

Table 2: **Experimental results**

	Uniform all rounds	Uniform rounds 11-20	Discr. all rounds	Discr. rounds 11-20
mean bids	145.10**	148.73**	105.81**	102.78**
mean higher bid	156.59**	158.65**	118.03**	114.79**
mean lower bid	133.61**	138.81**	93.59**	90.76**
mean seller profits	97.92**	110.51**	59.25**	64.64**
mean seller revenues	195.20**	199.20**	116.39**	103.07**
mean # transactions	1.35**	1.37**	1.05**	0.98**
mean seller cost	98.66	97.45	98.66	97.45

Note: ** significantly different from the respective other treatment for the same set of rounds at the 1% level of a two-sided MWU test.

form price auction also lie well within the predicted range. The predictions are even more accurate for the final 10 rounds. As predicted, mean bids are substantially higher in the uniform auction, which results in higher profits and revenues for sellers. Efficiency is also higher in the uniform auction as the average number of transactions per round is more than 33% higher for the uniform auction. All differences between the respective measures for our treatments are significantly different at the 1% level of a two–sided MWU test (see, e.g., Siegel and Castellan, 1988) taking each matching group of 6 subjects as an independent observation.

Furthermore, comparing the data from the last ten rounds to the theoretical predictions for the discriminatory auction (where we have a point prediction) with a Wilcoxon signed ranks test (cf. Conover, 1971, p. 206), we find that the predictions for bids, seller revenues, and number of transactions are not significantly different from the experimental data at any conventional significance level.

Figure 2 shows clear evidence of learning over time. Bids in the discriminatory auctions converge to the theoretical prediction of 100 while bids in the uniform auction start slightly below the predicted range but increase from there.

Figures 3 and 4 take a closer look at the distribution of individual bids,

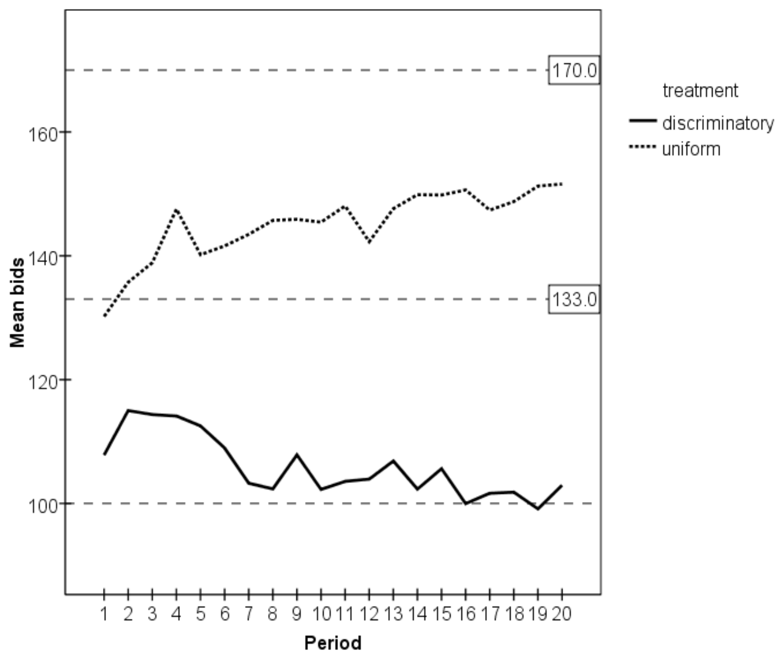

Figure 2: Time path of mean bids in the discriminatory and the uniform auction over the 20 rounds of the experiment.

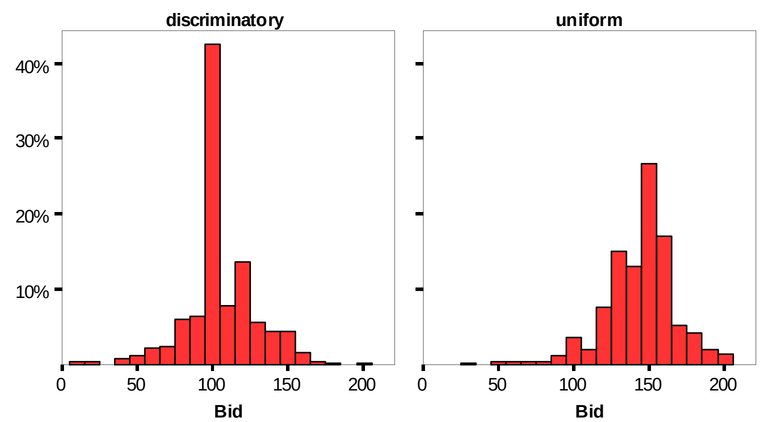

Figure 3: Distribution of individual bids in the discriminatory auction (left panel) and the uniform auction (right panel).

and pairs of bids, respectively. The left panel of Figure 3 shows a histogram of bids in the discriminatory auction. More than 40% of bids lie in the bracket containing the unique Nash equilibrium at 100. In fact, 37.6% of bids are concentrated exactly on 100.0 and 19.4% of subjects in the discriminatory auction chose the unique equilibrium quantity in *each* period.

In contrast, the right panel shows bids in the uniform auction with a mode at 150. Overall, in the discriminatory auction only 11% of bids lie above 133, while in the uniform auction 75% of bids lie above this threshold. Of all bids in the uniform auction, 65.3% are in the rationalizable range between 133.3 and 170.

When we consider the pairs of bids that were actually (randomly) matched in the experiment, we get Figure 4.[10] Bids in the discriminatory auction are scattered around 100. Only three pairs of bids are jointly above 133.3. Bids in the uniform auction are predominantly in the area of rationalizable

[10] Note that the matching for the scatter diagram is random and therefore somewhat arbitrary. Nevertheless, these are the outcomes subjects observed in the experiment after each period.

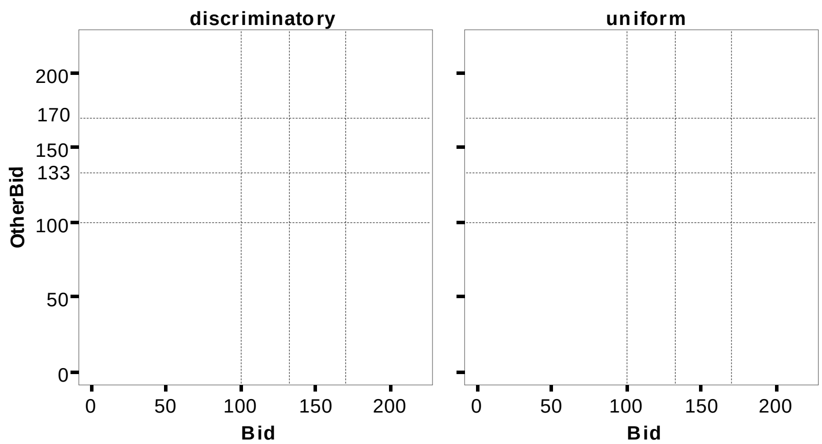

Figure 4: Scatter diagram of pairs of bids (as matched in the experiment) for the discriminatory auction (left panel) and the uniform auction (right panel).

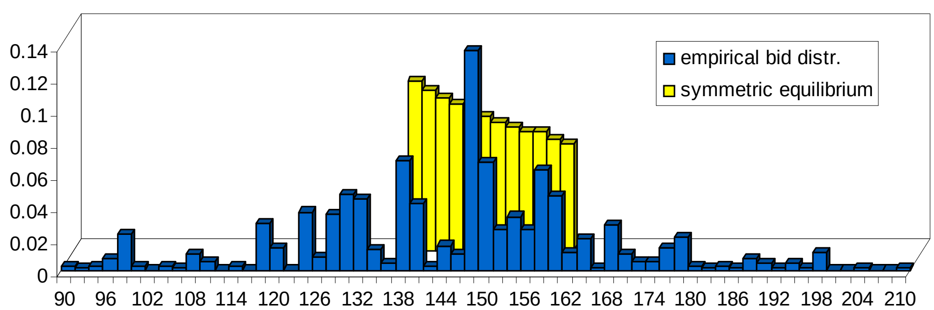

Figure 5: Distribution of bids in the uniform auction compared to the distribution of bids in the unique symmetric mixed strategy equilibrium.
Note: Empirical bid distribution is truncated at 90 and 210. The grid size is $0.01v = 2$.

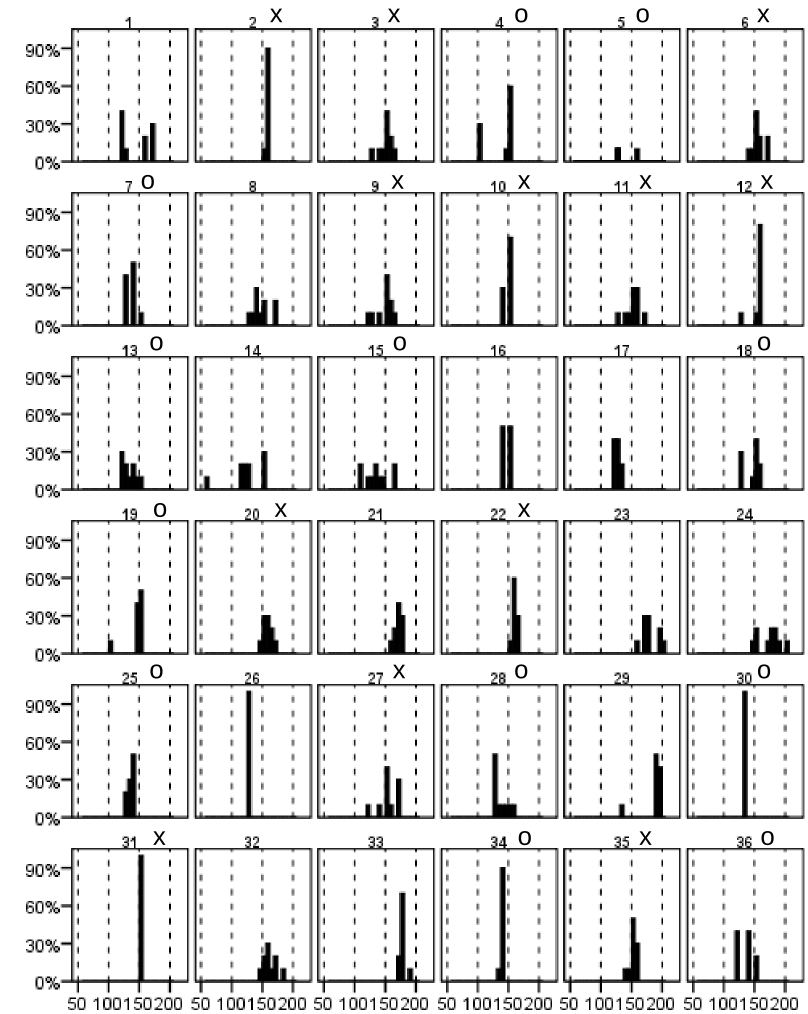

Figure 6: Distribution of bids for each individual subject in the uniform price treatment, periods 11-20.

Note: "o" denotes a distribution of individual bids that is not significantly different from the asymmetric equilibrium quantity 138 even at the 10% level according to an OLS fixed effects regression $b_{it} - 138 = \alpha_i + \varepsilon_{it}$. "x" denotes the same for 154.

strategies, the square between the lines at 133.3 and 170.

It should also be interesting to see how close the data are to the various equilibria of the uniform auction. Figure 5 compares the distribution of bids in the uniform auction treatment to the symmetric mixed strategy equilibrium distribution, both plotted for a grid size of $0.01v = 2$. Although the support of the mixed strategy describes fairly well the empirical bid distribution, the frequencies of the bids in the support do not match very well. This may be partly due to prominent number effects (bids of 150 are obviously very frequent) but it may also be a sign that the mixed strategy equilibrium does not describe very well what subjects actually do.

Subjects' individual behavior can be seen in Figure 6 which shows for each subject in the uniform price treatment the bid distribution in the second half of the experiment (see Figure 10 in the Appendix for all periods). Given the multiplicity of equilibria, this allows to shed some more light on subjects' strategies. Note first that almost all bids lie in the range of rationalizable strategies $[133.3, 170]$. However, while some subjects seem to mix over a broader range of strategies, others seem to play pure strategies, which, in fact, are often fairly close to the asymmetric pure equilibrium strategies of 138 and 154. To test this more rigorously, we run OLS regressions with subject fixed effects, $b_{it} - b^* = \alpha_i + \varepsilon_{it}$, $b^* \in \{138, 154\}$ with the null hypothesis that $\alpha_i = 0$. The results are indicated in Figure 6 by symbols "o" and "x" for 138 and 154, respectively. We cannot reject the hypothesis that $\alpha_i = 0$ for $b^* = 138$, even at the 10% level, for 12 subjects. For another 12 subjects, we cannot reject the hypothesis that $\alpha_i = 0$ for $b^* = 154$.

Finally, we asked subjects in the post–experimental questionnaire how much experience they had with auctions (e.g. eBay). We suspected that subjects with a lot of auction experience might show a bidding behavior that is closer to equilibrium behavior. In fact, a large number of subjects had considerable experience. The average number of auctions in which subjects had participated was 37.1 with a maximum of 300.

However, regressing auction experience together with treatment dummies on bids did not produce any significant coefficients for auction experience. The reason for this result may be that bidding behavior is already so close to equilibrium or rationalizable behavior that experience with auctions cannot make a difference.

5 Conclusion

In this paper, we report results of an experiment that compares uniform price auctions to discriminatory auctions in a setting with variable supply. The experimental results are quite close to the theoretical predictions. Just as theory predicts, bids are substantially higher in the uniform auction, and, consequently, so are revenues and profits for the seller. Despite the fact that in the discriminatory auction the seller never rejects bids that are above his marginal cost, the number of transactions is higher in the uniform auction, which implies that the uniform auction is also the more efficient auction in this setting.

The theoretical results hold under somewhat restrictive assumptions. It would be desirable to relax some of those assumptions in future work, in particular, the assumption that both bidders have the same valuation v. We see this assumption mainly as a technical simplification as does much of the literature (see Back and Zender, 2001; McAdams, 2007). We were able to show that allowing for some asymmetries with respect to bidders' values does not change the main results but future research needs to extend those results. It would also be interesting to conduct experiments with asymmetries in bidders' values.

Another interesting extension would be to allow for multi–unit demand by bidders. While some progress has been made in the literature,[11] this problems remains a challenge for future research.

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Appendix A: Proofs

Iterative elimination of strictly dominated strategies

Proof of Proposition 2. In order to simplify notation, let $v = 1$ in this proof without loss of generality. Note that in this case $\Delta \leq 0.01$. Applying the uniform distribution of costs, the expected payoff function (1) becomes

$$R_1^U(b_1, b_2) \tag{2}$$

$$= \begin{cases} (1-b_1)\min\{b_1, 1\} & \text{for} \quad 2b_2 \le b_1 & \text{Case } K \\ \begin{array}{c}(1-b_2)\min\{2b_2 - b_1, 1\} \\ +(1-b_1)\min\{2b_1 - 2b_2, 1\}\end{array} & \text{for} \quad b_2 \le b_1 < 2b_2 & \text{Case } L \\ (1-b_1)\min\{2b_1 - b_2, 1\} & \text{for} \quad b_2/2 \le b_1 < b_2 & \text{Case } M \\ 0 & \text{for} \quad b_1 < b_2/2 & \text{Case } N \end{cases}$$

Case K concerns the situation in which bidder 2 is never served, and bidder 1 pays his bid if served. The payoff in Case L applies to the scenario in which both bidders are served with a positive probability, but bidder 1 places a bid weakly above that of bidder 2. The payoff in Case M applies to the scenario in which bidder 1 submits the lower bid, but is still served with a positive probability. In Case N the payoff of bidder 1 is zero because his bid is so low that he is never served.

We start with the assertion that bidding above the valuation of 1 is strictly dominated by bidding 1, which will allow us to simplify the expected payoff function. We need to show that for all elements of the grid $b_2 \in B$ and all $b_1 > 1$ the inequality

$$R_1^U(1, b_2) > R_1^U(b_1, b_2)$$

holds. Indeed, it is easy to see that in Cases K and M the expected payoff is negative if $b_1 > 1$ and is zero if $b_1 = 1$. Case N does not apply when $b_1 \ge 1$ and $b_2 \in B$. It remains to consider Case L.

When $b_2 < 1$, the lower bid is clearly b_2, and the above inequality is equivalent to

$$(1-b_2)\min\{2b_2-1, 1\} > (1-b_2)\min\{2b_2-b_1, 1\} + (1-b_1)\min\{2b_1-2b_2, 1\}.$$

The left–hand side (LHS) gives the expected payoff of buyer 1 when this buyer bids 1, and the right–hand side (RHS) gives the expected payoff when this buyer bids $b_1 > 1$. For $b_2 < 1$ we have $1 - b_2 > 0$, $\min\{2b_2 - 1, 1\} = 2b_2 - 1$, and $\min\{2b_2 - b_1, 1\} = 2b_2 - b_1$. Therefore the term on the LHS is positive and larger than the first term on the RHS. The second term on the RHS is strictly negative. The inequality $LHS > RHS$ follows.

When $b_2 \geq 1$, we obtain $R_1^U(1, b_2) = 0$ because bidder 1 will always have to pay an amount equal to his valuation when served. The expected payoff $R_1^U(b_1, b_2)$ for $b_1 > 1$ depends in general on whether b_1 is the higher or the lower bid. Yet, in case that $b_2 > 1$ it is evident that $R_1^U(b_1, b_2) < 0$ because bidder 1 will always have to pay an amount higher than his valuation (either b_1 or b_2) when served. In case that $b_2 = 1$ we obtain $R_1^U(b_1, b_2) = (1 - b_1)(2b_1 - 2b_2) < 0$. Hence, $R_1^U(1, b_2) > R_1^U(b_1, b_2)$, which establishes the desired result.

This result is quite intuitive. If $b_1 > 1$ and the price bidder 1 pays equals b_1, he is clearly better off lowering his bid. That way he will lower the price he pays and possibly the probability of winning an item, both of which increases his payoff (the bidder prefers not to win if he has to pay a price higher than his valuation). Alternatively, the price bidder 1 pays can be determined by b_2, when $b_2 < b_1$. This happens with a probability of $2b_2 - b_1$. Thus, lowering b_1 increases the probability that bidder 1 pays b_2 instead of b_1. This makes bidder 1 better off.

After eliminating the strictly dominated bids $b_i > 1$ for both players, the min operators in (2) can now be replaced by their first argument. In order to identify the areas in which the expected payoff of bidder 1 increases in his own bid, we will consider the slope of the continuous function $R_1^U(b_1, b_2)$ as measured by the partial derivative with respect to b_1.

For this partial derivative we obtain

$$\frac{\partial R_1^U(b_1, b_2)}{\partial b_1} = \begin{cases} 1 - 2b_1 & \text{for} \quad 2b_2 \leq b_1 & \text{Case } K \\ 3b_2 + 1 - 4b_1 & \text{for} \quad b_2 \leq b_1 < 2b_2 & \text{Case } L \\ 2 + b_2 - 4b_1 & \text{for} \quad b_2/2 \leq b_1 < b_2 & \text{Case } M \\ 0 & \text{for} \quad b_1 < b_2/2 & \text{Case } N \end{cases} \tag{3}$$

Hence, the expected payoff of bidder 1 is increasing in his own bid in the following areas on the grid: in Case K for $b_1 < 1/2$, in Case L for

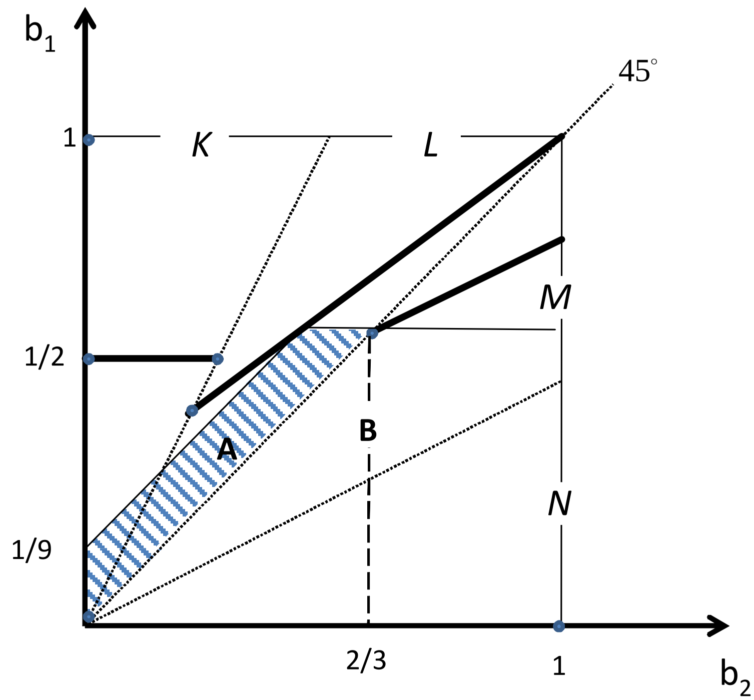

Figure 7: The expected payoff of bidder 1 strictly increases in b_1 in the areas below the solid lines of regions K, L and M. In particular, the expected payoff of bidder 1 increases in b_1 in the area $A \cup B$.

$b_1 < (3b_2 + 1)/4$, and in Case M for $b_1 < \min\{b_2, (2 + b_2)/4\}$. In Case N the expected payoff is zero. These areas are illustrated in Figure 7. The three rays from the origin divide the unit square into regions K, L, M, and N. The expected payoff of bidder 1 is strictly increasing in b_1 in the areas below the solid lines of the regions K, L, and M.

In particular, we can define area $A \cup B$ in Figure 7 given by the points on the grid $b_2 \in [0, 1]$ and $b_1 \in \left(b_2/2, \min\{b_2 + \frac{1}{9}, 2/3\}\right)$ for which the expected payoff $R_1^U(b_1, b_2)$ is strictly increasing in b_1. Furthermore, for $b_1 < b_2/2$ we have $R_1^U(b_1, b_2) = 0$ (see region N in Figure 7).

These preliminaries allow us to iteratively eliminate strictly dominated strategies. We will first eliminate in six steps the prices on the grid that fall

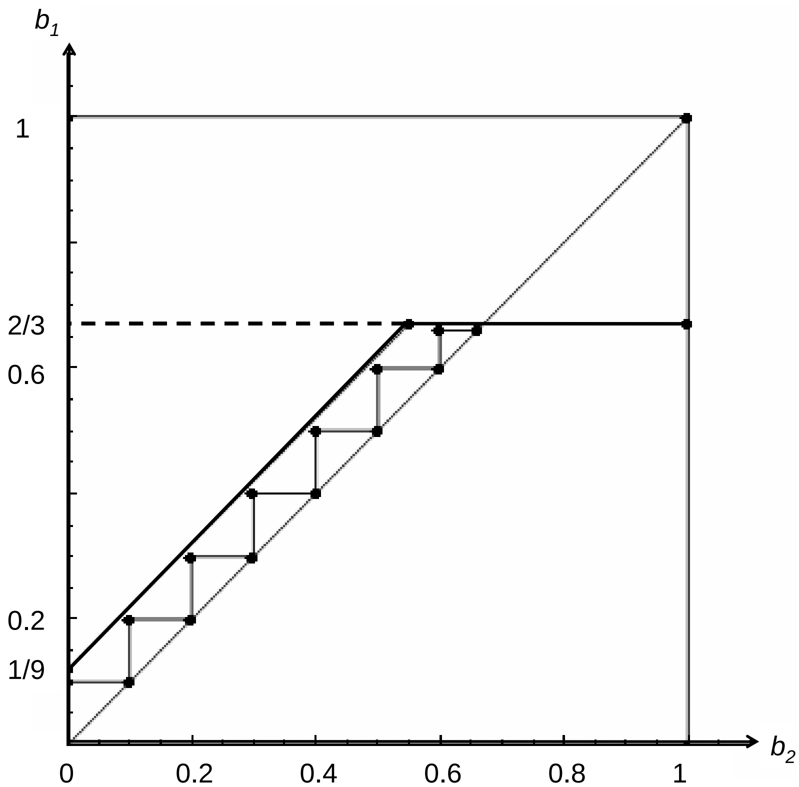

Figure 8: Seven steps of elimination of non-rationalizable strategies in the uniform price auction.

into the intervals $[0, 0.10)$, $[0.10, 0.20)$,..., $[0.50, 0.60)$, and finally the grid prices from the interval $[0.60, 0.66)$ (see Figure 8 for an illustration).

We note that the grid prices from the interval $[0, 0.10)$ are not best responses to any (mixed) strategy of bidder 2.[12] Indeed, if bidder 2 plays a mixed strategy which places a positive probability on bids from the interval $[0, 0.20)$, then bidding 0.10 will lead to a strictly higher payoff for bidder 1 than using any of the bids from the interval $[0, 0.10)$. On the other hand, if bidder 2 plays a mixed strategy which places a zero probability on all

[12] In finite two-player games, the property of "never a best response" is equivalent to being strictly dominated.

the bids lying in the interval $[0, 0.20)$, then any strategy $b_1 \in [0, 0.10)$ leads to a zero expected payoff for bidder 1. In this case, submitting a bid of, for instance, 3/4, would guarantee a positive payoff (bidder 2 does not bid above 1). Thus, grid prices from the interval $[0, 0.10)$ can be eliminated as not being a best response to any strategy of bidder 2, i.e. non-rationalizable. As we verified that there are no rationalizable bids below 0.10 for each player, we can perform a second step of elimination with respect to the prices on the grid from the interval $[0.10, 0.20)$. Playing 0.20 guarantees a strictly higher payoff when bidder 2 uses a (mixed) strategy placing a positive probability on bids from the interval $[0.10, 0.40)$; and playing 3/4 guarantees a strictly higher payoff when bidder 2 uses a (mixed) strategy that does not place a positive probability on bids from that interval. The remaining iterations are analogous, and allow us to establish that all bid prices on the grid below 0.66 are not rationalizable.

Finally, direct inspection of the payoff function shows that $b_1 = 0.69$ strictly dominates all pure strategies from the interval $[0.85, 1]$ once strategies $b_2 > 1$ are eliminated. Using Gambit we find that no further pure strategies can be eliminated by any other strategy (pure or mixed). That is, all remaining strategies in the interval $[0.66, 0.84]$ are rationalizable. ■

Proof of Proposition 3. Let bidders have a (weakly) concave utility function $u(x)$ and let us normalize without loss of generality $u(0) = 0$. Let us assume that $\tilde{u}(x)$ is more risk averse than $u(x)$, that is $\tilde{u}(x) = \psi(u(x))$, and $\psi(\cdot)$ is increasing and strictly concave. We denote by b^D and $\tilde{b}^D$ the dominant strategies in the discriminatory auction of bidders with utility functions $u(x)$ and $\tilde{u}(x)$, respectively. Similarly, $\underline{b}^U$ and $\tilde{\underline{b}}^U$ denote the lower bounds of the rationalizable strategies in the uniform price auction of bidders with preferences $u(x)$ and $\tilde{u}(x)$, respectively. We will demonstrate that for a sufficiently large n, (i.e. sufficiently small grid size) the following inequalities hold: $\tilde{b}^D > b^D$ (the dominant strategy bid in the discriminatory auction is increasing in the degree of risk aversion); $\tilde{\underline{b}}^U > \underline{b}^U$(the lower bound on rationalizable bids in the uniform price auction is increasing in the degree of risk aversion); and $\underline{b}^U > b^D$ (the lower bound of rationalizable bids in the uniform price auction is higher than the dominant strategy of the discriminatory auction) when bidders are risk averse. To prove these statements we use the following lemmas, which are proven below.

Lemma 1 (Discriminatory) *The dominant strategy bid in the discriminatory auction b^D is given by the solution to the equation $b = u(v-b)/u'(v-b)$ if this solution belongs to the grid. Otherwise either the closest bid on the grid from below or the closest bid from above (or both) will be a dominant strategy.*

Lemma 2 (Uniform) *The lower bound on rationalizable bids in the uniform auction $\underline{b}^U$ is given by the solution to the equation $b/2 = u(v-b)/u'(v-b)$ if this solution belongs to the grid. Otherwise the closest bid on the grid from below determines this lower bound.*

Lemma 3 (Degree of risk aversion) *The inequality $\tilde{u}(v-b)/\tilde{u}'(v-b) > u(v-b)/u'(v-b)$ holds for all $b \in [0, v)$.*

Given these lemmas, the statements formulated in the proposition follow easily from Figure 9. Note that the function $u(v-b)$ is strictly decreasing in b and $u'(v-b)$ is strictly increasing in b because u is strictly concave. It follows that $u(v-b)/u'(v-b)$ is a strictly decreasing function as presented in Figure 9. In the risk–neutral case the ratio $u(v-b)/u'(v-b)$ equals $v-b$.

Lemma 3 asserts that the ratio $\tilde{u}(v-b)/\tilde{u}'(v-b)$ is higher than the ratio $u(v-b)/u'(v-b)$ for every $b \in [0, v)$ as shown in the figure. Thus, using the result in Lemma 1 and Figure 9 we conclude that $\tilde{b}^D > b^D \geq v/2$. Similarly, using Lemma 2 and the figure we observe that $\underline{\tilde{b}}^U > \underline{b}^U \geq 2v/3$. Finally, since the intersection of $u(v-b)/u'(v-b)$ with b must always be to the right of the intersection with $b/2$, we find that for a sufficiently fine grid $\underline{b}^U > b^D$. ■

Next we turn to the proof of the three lemmas.

Proof of Lemma 1 (Discriminatory). The expected payoff of bidder i is

$$R_i^D(b_1, b_2) = u(v - b_i)b_i/v.$$

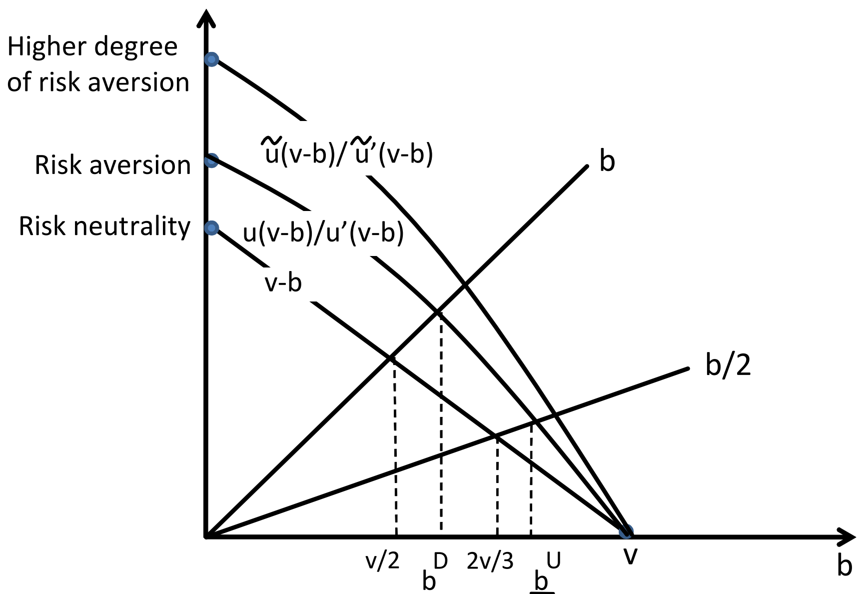

Figure 9: Comparison of bids according to the degree of risk aversion in the discriminatory and the uniform price auction.

In order to identify the areas in which the expected payoff of bidder 1 increases in his own bid, we consider the derivative of the continuous function $R_i^D(b_1, b_2)$. The first order condition is given by

$$-u'(v - b_i)b_i/v + u(v - b_i)/v = 0.$$

It is easily seen that the LHS of the first order condition is decreasing in b_i. Indeed, u is increasing and weakly concave, and therefore $-u'(v - b_i)b_i/v$ and $u(v - b_i)/v$ are decreasing in b_i. Hence, $R_i^D(b_1, b_2)$ has a unique maximizer. The dominant strategy b^D will be equal to this maximizer if this maximizer belongs to the grid. Otherwise the closest point on the grid from above or below (or both) will be a dominant strategy. ■

Proof of Lemma 2 (Uniform). In order to simplify notation, let $v = 1$ in this proof without loss of generality. We can claim again that bids $b_i > 1$ are strictly dominated by the bid $b_i = 1$. The proof is analogous to the one provided in Proposition 2, and therefore omitted here. To identify the areas in which the expected payoff function $R_1^U(b_1, b_2)$ increases, we consider the partial derivative of the continuous function

$$\frac{\partial R_1^U(b_1, b_2)}{\partial b_1} =$$

$$\begin{cases} u(1-b_1) - u'(1-b_1)b_1 & \text{for} \quad 2b_2 \le b_1 \quad \text{Case } K \\ -u(1-b_2) + 2u(1-b_1) \\ \quad -u'(1-b_1)(2b_1 - 2b_2) & \text{for} \quad b_2 \le b_1 < 2b_2 \quad \text{Case } L \\ -u'(1-b_1)(2b_1 - b_2) \\ \quad +2u(1-b_1) & \text{for} \quad b_2/2 \le b_1 < b_2 \quad \text{Case } M \\ 0 & \text{for} \quad b_1 < b_2/2 \quad \text{Case } N \end{cases} \tag{4}$$

We will demonstrate that for a large enough but finite n the strategies below $\underline{b}^U$ do not survive the iterated deletion of strictly dominated strategies. We first verify that bidding 0 is not a best response to any pure or mixed strategy of the other bidder, and this property allows us to claim that any mixed strategy which places a positive probability on the bid 0 is strictly dominated. By an iterative procedure we subsequently eliminate also all mixed strategies that place a positive probability on the bids Δ, $2\Delta, 3\Delta$, etc., which are below $\underline{b}^U$.

The following properties of (4) will be useful for our analysis.

Case K. Bidder 2 is never served, and the payoff of bidder 1 depends only on his own bid. The payoff of bidder 1 in this case is the same as the payoff in the discriminatory auction format. The optimal bid is b^D and for all $b_1 \in [0, b^D)$ we have $\frac{\partial R_1^U(b_1,b_2)}{\partial b_1} > 0$.

Case L. Note that the partial derivative from above on the 45°–line is

$$\left.\frac{\partial^+ R_1^U(b_1,b_2)}{\partial b_1}\right|_{b_1 = b_2} = u(1-b_1) > 0.$$

This partial derivative is continuous and strictly bounded away from zero in the interval $[0, \underline{b}^U]$. Therefore, when Δ is small enough and both bidders submit the same bid, it is profitable for bidder 1 to incrementally increase his bid. Formally, for $b_1 = b_2$ the inequality $R_1^U(b_1+\Delta, b_2) > R_1^U(b_1, b_2)$ holds.

Case M. Observe that

$$\frac{\partial R_1^U(b_1,b_2)}{\partial b_1} = -u'(1-b_1)(2b_1-b_2)+2u(1-b_1) > -u'(1-b_1)b_1+2u(1-b_1).$$

As $u(\cdot)$ is increasing and strictly concave, the expression $-u'(1-b_1)b_1+2u(1-b_1)$ is strictly decreasing in b_1 and is positive for $b_1 < \underline{b}^U$ (recall that $\underline{b}^U$ is the highest bid on the grid that does not exceed the solution of the equation $-u'(1-b_1)b_1 + 2u(1-b_1) = 0$). Thus, the inequality $\frac{\partial R_1^U(b_1,b_2)}{\partial b_1} > 0$ holds for all $b_1 \in [0, \underline{b}^U)$.

Case N. Bidder 1 is never served and his expected payoff is zero.

These preliminaries are sufficient to claim that mixed strategies placing a positive probability on any of the bids $0, \Delta, 2\Delta, ...$ below $\underline{b}^U$ are never a best response. Indeed, if b_2 is such that one of the Cases K, L or M applies, then it is profitable for bidder 1 to increase his bid by one increment. If Case N applies, the bid of bidder 1 is so low that he never wins an item. In this case any strategy that brings player 1 a positive expected payoff (e.g. the bid 3/4) will lead to a strictly higher payoff for player 1. Thus, strategy

0 is eliminated because either Δ or $3/4$ is a better response to any strategy of bidder 2. Strategy Δ is eliminated because either 2Δ or $3/4$ is a better response to any strategy of bidder 2. Strategy 2Δ is eliminated because either 3Δ or $3/4$ is a better response to bidder 2, etc. The desired result is established after a finite number of iterations. ■

Proof of Lemma 3 (Degree of risk aversion). Let $\tilde{u}(x) = \psi(u(x))$, $\psi(\cdot)$ is increasing and strictly concave, and $\psi(0) = 0$. We have to show that $\tilde{u}(v-b)/\tilde{u}'(v-b) > u(v-b)/u'(v-b)$ for all $b \in [0, v)$. With the substitution

$$x = v - b$$

the above inequality is equivalent to

$$\frac{\tilde{u}(x)}{\tilde{u}'(x)} > \frac{u(x)}{u'(x)}, \forall x \in (0, v].$$

Observe that

$$\begin{aligned} \frac{\tilde{u}(x)}{\tilde{u}'(x)} &= \frac{\psi(u)}{\psi'(u) \cdot u'(x)} > \frac{u(x)}{u'(x)} \Longleftrightarrow \\ \frac{\psi(u)}{\psi'(u)} &> u \Longleftrightarrow \psi'(u) < \frac{\int_0^u \psi'(s)ds}{u}. \end{aligned} \tag{5}$$

By the Intermediate Value Theorem of Integration there exists $z \in (0, u)$ such that

$$\int_0^u \psi'(s)ds = u\psi'(z).$$

Thus, inequality (5) follows since $z < u$ and $\psi(\cdot)$ is strictly concave by assumption. ■

Nash equilibria

From (3) and (2) we can derive the (pure) best response correspondence for the risk neutral case,

$$b_1^*(b_2) := \arg\max_{b_1} R_1^U(b_1, b_2) = \begin{cases} 0.5 & \text{for } b_2 < \frac{3-\sqrt{2}}{7}, \\ \{\frac{16-3\sqrt{2}}{28}, 0.5\} & \text{for } b_2 = \frac{3-\sqrt{2}}{7}, \\ \frac{3b_2+1}{4} & \text{for } b_2 \in \left(\frac{3-\sqrt{2}}{7}, 3/4\right), \\ \{\frac{11}{16}, \frac{13}{16}\} & \text{for } b_2 = 3/4, \\ \frac{b_2+2}{4} & \text{for } b_2 \in (3/4, 1.1]. \end{cases}$$

which is plotted in Figure 1. The two intersections form two asymmetric pure strategy equilibria at $(b_i, b_{-i}) = (0.77, 0.69)$, $i = 1, 2$.

In order to find mixed equilibria, we used Gambit with a strategy grid size 0.01. There are one symmetric mixed strategy equilibrium and 1246 asymmetric ones. The support of all Nash equilibria is contained in $[0.69, 0.81]$.

Appendix B: Translation of instructions

Welcome to our experiment. Please read these instructions carefully. They are the same for every participant. Please do not talk with other participants and remain quiet during the entire experiment. Please turn off your cell phone and don't switch it on until the end of the experiment. If you have any question, raise your hand and we will come to you.

The experiment will consist of twenty rounds. An auction with two bidders will take place in each round. You are one bidder, the second one is chosen randomly for each round. So you are bidding with somebody else in each round.

All money during this experiment is real money. At the end of the experiment all the money in your account will be paid to you at the rate 100 cent = 1 €. At the beginning of the experiment you are endowed with 400 cent.

Auction rules

Each round consists of one auction. During this auction the computer can produce and sell 0, 1 or 2 units of the same good. Each unit of this good is worth 200 cent for a bidder.

The computer has to pay production cost for each unit produced. This production cost is the same for each unit produced in one round and is chosen randomly for each round. Each number between 0 and 200 (with one decimal place) has the same probability of being chosen, so each production cost of the amount {0; 0.1; 0.2; ... ; 199.9; 200.0} has the same probability of being a certain round‘s production cost. The costs of the different rounds are totally independent of each other.

During each round the auction proceeds according to the following rules:

- Each bidder makes an offer.
- All bids from 0 to 220 cent are possible, with one decimal place {0; 0.1; 0.2; ... ; 219.9; 220.0}.
- Each bidder can buy no more than one unit in each round.
- The computer can choose,
  - either to produce 0 units and not to sell anything to anybody,
  - or to produce 1 unit and to sell it for the higher bid price to the bidder offering that price,
  - or to produce 2 units and sell one to each bidder for the lower bid price offered.
- The computer always chooses the option that maximizes its profit. Its profit equals its revenue from selling the units minus its production cost. If two possibilities yield the same profit, the computer chooses the one with more units sold.

Here are some examples:

You are bidding 190 cent, the other bidder is willing to pay only 140 cent. Production cost turns out to be 80 cent per unit in this round. The computer makes the following calculation:

- If it produces and sells one unit at a price of 190 cent, it will make a profit of: 190 cent − 80 cent = 110 cent.

- If it produces two units and sells one to each bidder at a price of 140 cent (it can just demand the lower offer according to the rules), it will yield a profit of 140 cent minus 80 cent per unit, this means a total profit of: 2 × (140 cent − 80 cent) = 120 cent.

This example shows that the computer will be better off if it produces two units and sells them at a price of 140 cent. So that is what it is going to do.

You receive 200 cent per unit you purchased during the auction at the end of the round. So for you this round's payoff is

- 200 cent minus the price you paid for the good if you get one unit,
- 0 if you receive no unit.

So in the above example you would have made a profit of

- 200 cent − 140 cent = 60 cent;
- the same is true for the other bidder.

At the end of each round you are told the value of the two bids, the amount of units sold (0, 1 or 2), this round's payoffs (yours and the second bidder's), the production cost and the balance of your account. Please write down all these data on your record sheet.

After this, a new round starts. Bidders then are matched and the production costs are chosen randomly again for each round. All rounds are independent of each other.

At the end of round twenty, please answer a short questionnaire that appears on your computer screen. After answering the questionnaire, please come to the instructors and bring your record sheet. All the money in your account will be paid to you in cash.

That's all about the rules. They are the same for every participant. If you have any question, raise your hand and we will come to you. Before starting the experiment, please answer the questions on the following page to make sure that you understood all the rules.

Test questions

1. If the price were 23 cents and the computer allocated one unit to you, what would be your payoff for this period?
2. If the price were 210 cents and the computer allocated one unit to you, what would be your payoff for this period?
3. If the price were 23 cents and the computer did not allocate a unit to you, what would be your payoff for this period?

   Assume that production cost per unit were 60 cents. Your bid is 150 cents and the other bidder bids 120 cents.

4. Calculate the computer's profit if it sells 1 unit _____ and if it sells 2 units _____.
5. How many units would the computer allocate and to whom?
6. What would be your payoff and the payoff of the other bidder when the computer chooses its better alternative?

   Assume that production cost per unit were 110 cents. Your bid is still 150 cents and the other bidder bids 120 cents.

7. Calculate the computer's profit if it sells 1 unit _____ and if it sells 2 units _____.
8. How many units would the computer allocate and to whom?
9. What would be your payoff and the payoff of the other bidder when the computer chooses its better alternative?

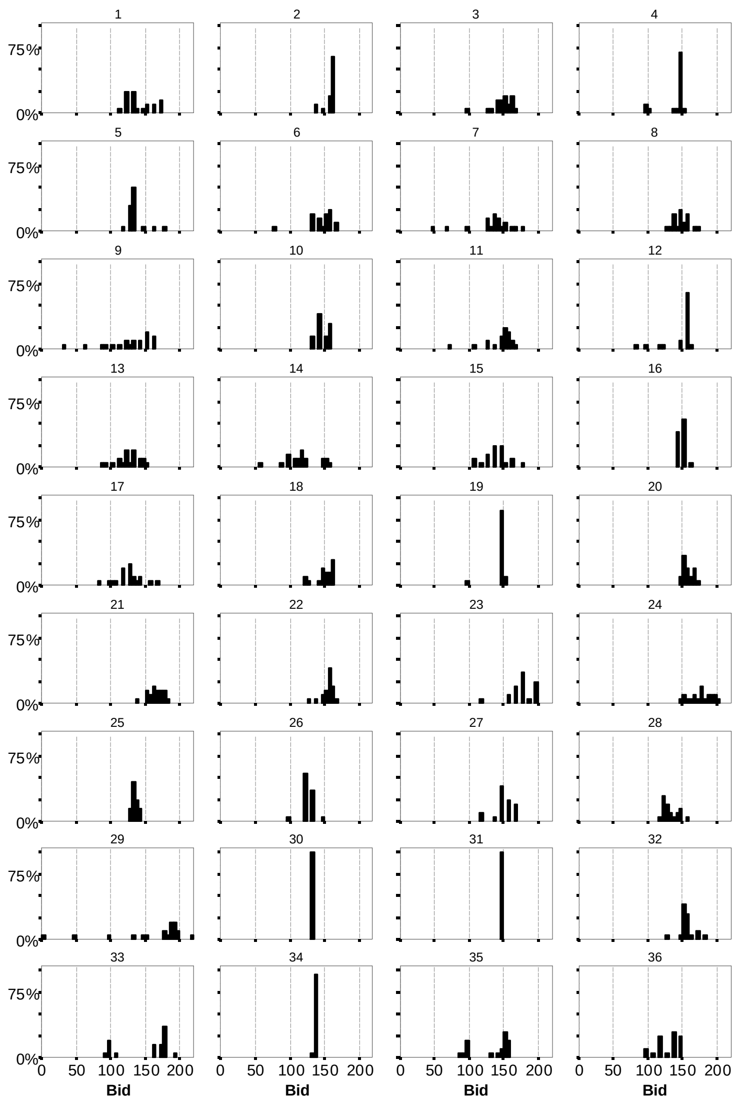

Figure 10: Distribution of bids for each individual subject in the uniform price treatment, all periods.